Notes on Economics of Banking

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Contents

1 Chapter 1 – Introduction

1.1 What is a bank?

First operational definition: *A bank is an institution whose current operations consist in granting loans and receiving deposits from the public.*
Contemporary banking theory divides banking functions into four main categories:

1. Offering access to a payment system
    - Money changing (e.g. currencies).
    - Management of deposits.
    - Payment services – financial infrastructure.
2. Transforming assets
    - Convenience of denomination.
    - Quality transformation.
    - Maturity transformation.
3. Managing risks
    - Credit risk.
    - Interest rate risk.
    - Liquidity risk.
    - (Off-balance sheet items).
4. Processing information and monitoring borrowers
    - Managing problems resulting from imperfect information on borrowers.

1.2 Banking in General Equilibrium theory

Model: A two-period model with one good. Three agents in the economy: The consumer, the firm and the bank.

- **The Consumer** chooses consumption profile in order to maximize utility under the budget constraints:

$$C_1 + B_h + D^+ = \omega_1$$

$$C_2 = \pi_f + \pi_b + (1 + r)B_h + (1 + r_D)D^+,$$

where D^+ denotes deposits and B_h is amount of securities held. In order for the consumer to hold both deposits and securities, it must hold that $r = r_D$.

- **The Firm** chooses its investment level and financing; bank credit L and issuance of securities B_f in order to maximize its profits under the constraints:

$$\pi_f = f(I) - (1+r)B_f - (1+r_L)L^-$$

$$I = B_f + L^-$$

  In order for the firm to be willing to both taking bank loans and issue securities, it must hold that $r = r_L$.

- **The bank** seeks to maximize its profits under the constraints:

$$\pi_b = r_L L^+ - rB_b - r_D D^-$$

$$L^+ = B_b + D^-$$

In general equilibrium (1) each agent behaves optimally, and (2) each markets clear. In order for all markets to be clearing (functioning at all) it must be that $r = r_L = r_D$. Hence, **there is no role for banks in general equilibrium since it makes zero profits and have no impact on other agents**. This facts is due to securities and loans / deposits being perfect substitutes.
This brings about a need for another theory of the existence of banks. This is the scope of the next chapters.

2 Chapter 2 – Why banks?

Chapter 2 assess the existence of banks as a result of transaction costs, which in turn are caused by information asymmetry. Direct transaction costs are not considered to be important as of today. Banks can be seen as coalitions of individuals who exploit economies of scale or economies of scope in transaction technology. The chapter contains three examples showing different reasons for asymmetric information leading to existence of transaction costs.

2.1 Liquidity insurance

Consider a three-period, one good economy with a continuum of ex-ante identical agents, each endowed with one unit of the good. Ex-post, at $t = 1$, the agents learn whether they have the utility function $u(C_1)$ or $\rho u(C_2)$. The ex-ante expected utility is thus

$$U = \pi_1 u(C_1^1) + \pi_2 \rho u(C_2^2), \tag{1}$$

where C_t^i is the consumption of an agent of type i at time t. It is assumed that $u' > 0$ and $u'' \leq 0$. The good can either be stored (with a perfect storage technology) or invested in a long-run technology which returns $R > 1$ units at $t = 2$, but only $L < 1$ if liquidated at $t = 1$.

- **Autarky**. The agent can here consume either

$$C_1 = 1 - I + LI = 1 - I(1 - L) \leq 1, \tag{2}$$

if he has to consume early, or

$$C_2 = 1 - I + RI = 1 + I(R - 1) \leq R. \tag{3}$$

- **Market economy**. We now allow trade, in the sense of a bond market at $t = 1$. The price of a bond which returns 1 unit of the good at $t = 2$ is p. The agent can then obtain

$$C_1 = 1 - I + pRI,$$

or if the agent consumes at $t = 2$:

$$C_2 = \frac{1 - I}{p} + RI = \frac{1}{p}[1 - I + pRI]$$

In equilibrium it must be that $p = \frac{1}{R}$, leading to $C_1 = 1$ and $C_2 = R$. Hence, **market equilibrium Pareto dominates the autarky solution**.

- **Pareto efficient allocation**. We now seek to characterize the Pareto optimal solution, specifically in order to decide whether the market reaches this solution. The Pareto Optimal allocation satisfies:

$$\max \pi_1 u(C_1) + \rho\pi_2 u(C_2)$$
$$\text{s.t.} \quad \pi_1 C_1 + \pi_2 \frac{C_2}{R} = 1$$

The first-order condition for the optimal allocation is [1]

$$u'(C_1^*) = \rho R u'(C_2^*) \tag{4}$$

Then we can conclude that the market allocation will only be Pareto efficient in the peculiar case where $u'(1) = \rho R u'(R)$. We conclude the following

Proposition 2.1 *A market economy does not provide perfect insurance against liquidity shocks, and therefore does not lead to an efficient allocation of resources. The efficient allocation can be reached by a financial intermediary (bank) through the introduction of a deposit contract.*

[1]Can be derived through the usual MRS-condition.

2.2 Information sharing coalitions

This theory explores the role of a bank as a coalition of borrowers. We study a model, where there exists a large number of entrepreneurs, each with a risky project. The project requires a fixed investment of 1, and yields a net return of $\tilde{R}(\theta) \sim N(\theta, \sigma^2)$, where σ^2 is the same for all projects, and is at the same time common knowledge. In opposition, θ is private information to the entrepreneur, and only the statistical distribution of θ is common knowledge. If θ was observable, the entrepreneur would sell the project at a price $P(\theta) = E[\tilde{R}(\theta)] = \theta$, giving the entrepreneur a total (secure) wealth $W_0 + \theta$.
The entrepreneur is risk averse, $u(\omega) = -e^{-\rho\omega}$. By self-financing the project, the entrepreneur's expected utility will be

$$E[u(W_0 + \tilde{R}(\theta))] = u(W_0 + \theta - \frac{1}{2}\rho\sigma^2). \tag{5}$$

Therefore, an entrepreneur will only sell the project if

$$W_0 + P > W_0 + \theta - \frac{1}{2}\rho\sigma^2 \Leftrightarrow \theta < \hat{\theta} = P + \frac{1}{2}\rho\sigma^2 \tag{6}$$

We can from the expression above identify the adverse selection problem, namely that only entrepreneurs with low expected return will issue equity.
Assume now that θ can take on only one of two possible values $\theta_1 < \theta_2$. If both types are marketed then it must be that

$$P = \pi_1\theta_1 + \pi_2\theta_2 \quad \wedge \quad P \geq \theta_2 - \frac{1}{2}\rho\sigma^2$$

Combining these two expressions yields

$$\pi_1(\theta_2 - \theta_1) \leq \frac{1}{2}\rho\sigma^2 \tag{7}$$

If (7) is satisfied, there will exist a pooling equilibrium in which all projects are financed, since the risk premium here outweighs the adverse selection effect. **If** (7) **is not satisfied, there will be a separating equilibrium in which good projects are self-financed. Here the equilibrium will be inefficient**.

In the signalling equilibrium the good entrepreneurs will prefer self-financing, but would like to stick with partial self-financing if possible. If they can convince investors that their project is of good quality such a solution will become available. **Leland & Pyle (1977)** show that this solution can be achieved through a self-financing fraction α. The condition is the IC_1-condition:

$$\begin{aligned} u(W_0+\theta_1) \geq E[u(W_0+(1-\alpha)\theta_2+\alpha\tilde{R}(\theta_1))] \Rightarrow \\ \theta_1 \geq (1-\alpha)\theta_2+\alpha\theta_1-\frac{1}{2}\rho\sigma^2\alpha^2 \Leftrightarrow \\ \frac{\alpha^2}{1-\alpha} \geq \frac{2(\theta_2-\theta_1)}{\rho\sigma^2}. \end{aligned} \tag{8}$$

Proposition 2.2 (Leland & Pyle (1977)) *There is a continuum of equilibria all satisfying inequality* (8) *characterized by a low price of equity $P_1 = \theta_1$ for projects* without *self-financing and a high price of equity $P_2 = \theta_2$ for projects* with *self-financing.*

Of course, the best of these possible solutions will be the one with the lowest possible α, leading (8) to hold with equality. Define then the cost of capital as

$$C(\sigma) = \frac{1}{2}\rho\sigma^2\alpha(\sigma), \tag{9}$$

where $\alpha(\sigma)$ is defined implicitly from (8) holding with equality. The following result is obtained

Proposition 2.3 Diamond (1984) *The unit cost of capital decreases with the size of the coalition of borrowers.* $\left(\frac{\partial C(\sigma)}{\partial\sigma} > 0\right)$

2.3 Delegated Monitoring

This branch of the asymmetric information explanation of the existence of financial intermediaries takes a view of the latter as credit monitors.

In the model of Diamond (1984) there are n identical borrowers, that can each be monitored at a cost K or the moral hazard problem can be solved by a debt contract at a non-pecuniary cost C. Assume that m investors are needed in order to finance one project and that there are at least nm investors. Hence, the cost of direct monitoring is nmK and with delegated monitoring $nK + C_n$, where C_n is the cost of monitoring the monitor. Diamond states the following result:

Proposition 2.4 (Diamond (1984)) *If monitoring is efficient* ($K < C$)*, investors are small* ($m > 1$) *and investment is profitable, delegated monitoring dominates direct lending as soon as n is large enough* ($nK + C_n < nmK$)*.*

This proposition has been criticized for the use of a non-pecuniary penalty (C).

2.4 Coexistence of banks and direct lending

This part of the book holds a few models on this issue, here only one of the conclusions is stated.

Proposition 2.5 Holmström & Tirole (1993) *In equilibrium, only well capitalized firms* ($A \geq \bar{A}$) *can issue direct debt. Reasonably capitalized firms* ($\underline{A}(\beta) \leq A < \bar{A}$) *borrow from banks and undercapitalized firms* $A \leq \underline{A}(\beta)$) *cannot invest.*

3 Chapter 3 – The IO approach

The chapter compares the equilibria obtained in the banking business under different market conditions. The entire chapter views banking as the production of deposit and loans services, the bank's cost function being $C(D, L)$. We define α to be the fraction of deposit to be held by bank in the form of cash reserves.

3.1 Perfect Competition

The chapter starts with a story on perfect competition. The bank's profit will be

$$\pi = r_L L + rM + r_D D - C(D, L),$$

where $M = (1-\alpha)D - L$ is the net position of the bank on the interbank market, leading the profit to be rewritten as

$$\pi(D, L) = (r_L - r)L + (r(1-\alpha) - r_D)D - C(D, L) \tag{10}$$

As usual, the profit is maximized by taking the first-order conditions:

$$\begin{aligned} \frac{\partial \pi}{\partial L} &= (r_L - r) - \frac{\partial C}{\partial L}(D, L) = 0 \\ \frac{\partial \pi}{\partial D} &= (r(1-\alpha) - r_D) - \frac{\partial C}{\partial D}(D, L) = 0 \end{aligned} \tag{11}$$

From the above expressions we conclude the following:

Proposition 3.1 *A competitive bank will set the volume of loans and deposits such that the intermediation margins equal the marginal costs* ($p = MC$)*. Furthermore,* $r_D \uparrow \;\Rightarrow\; D \downarrow$ *and* $r_L \uparrow \;\Rightarrow\; L \uparrow$*. The cross-effects depend on the sign of* $\frac{\partial^2 C}{\partial L \partial D}$*. If* $\frac{\partial^2 C}{\partial L \partial D} < 0$ *there are economics of scope.*

3.2 Monopoly (Monti-Klein)

The Monti-Klein model consider a monopolistic bank confronted with a downward sloping demand $L(r_L)$ and an upward sloping supply $D(r_D)$. The decision variables are D and L.

The firm seeks to maximize profits:

$$\pi(L,D) = [r_L(L) - r]\,L + [r(1-\alpha) - r_D(D)]\,D - C(D,L) \tag{12}$$

The corresponding first-order conditions are

$$\begin{aligned} \frac{\partial \pi}{\partial L} &= r'_L(L)L + r_L - r - \frac{\partial C}{\partial L}(D,L) = 0 \\ \frac{\partial \pi}{\partial D} &= -r'_D(D)D + r(1-\alpha) - r_D - \frac{\partial C}{\partial D}(D,L) = 0 \end{aligned} \tag{13}$$

These can be rewritten to be expressed in terms of Lerner indices:

$$\begin{aligned} \frac{r_L^* - (r + C'_L)}{r_L^*} &= \frac{1}{\epsilon_L(r_L^*)} \\ \frac{r(1-\alpha) - C'_D - r_D^*}{r_D^*} &= \frac{1}{\epsilon_D(r_D^*)} \end{aligned} \tag{14}$$

We get the following result

Proposition 3.2 *If management costs are separable, the optimal deposit rate is independent of the loan market and vice versa. If* r *increases, both* r_L^* *and* r_D^* *increases.*

3.3 Cournot Oligopoly

We now try to use the Monti-Klein setup for a Cournot Oligopoly model of banking. With the cost function $C(D,L) = \gamma_D D + \gamma_L L$, the bank's profit can be stated as

$$\pi(D_n, L_n) = \left[r_L \left(L_n + \sum_{m \neq n} L_m^* \right) - r \right] L_n + \left[r(1-\alpha) - r_D \left(D_n + \sum_{m \neq n} D_m^* \right) \right] D_n - C(D_n, L_n) \tag{15}$$

As with the monopoly case we can take first-order conditions and rewrite them in terms of Lerner indices:

$$\begin{aligned} \frac{r_L^* - (r + \gamma_L)}{r_L^*} &= \frac{1}{N\epsilon_L(r_L^*)} \\ \frac{r(1-\alpha) - \gamma_D - r_D^*}{r_D^*} &= \frac{1}{N\epsilon_D(r_D^*)} \end{aligned} \tag{16}$$

We see that results are similar to the ones in the monopoly case except from the N in the denominator. The monopoly case is thus just a special case of the Cournot model and perfect competition is also just a special case of the Cournot model (as usual). This makes our lives a lot easier since we can use the Monti-Klein model widely.

3.3.1 Deposit rate regulations

We now apply the Monti-Klein model to the case where we impose a deposit rate ceiling $r_D \leq \bar{r}_D$. Such a policy has the intention of lowering r_L (expansionary policy). The question is whether such a policy is effective.
Clearly, if $r_D^* \leq \bar{r}_D$, the interest rate ceiling has no effect. The more interesting case is when the opposite is the case. Then it must now hold that

$$\frac{\partial \pi}{\partial r_L}(\bar{r}_D, r_L) = 0$$

We want to determine the sign of $\frac{dr_L}{d\bar{r}_D}$. In order to do this we invoke the implicit function theorem:

$$\frac{\partial^2 \pi}{\partial r_L^2}\frac{dr_L}{d\bar{r}_D} + \frac{\partial^2 \pi}{\partial r_L \partial r_D} = 0 \tag{17}$$

Since $\frac{\partial^2 \pi}{\partial r_L^2} < 0$, it must be that sign $\left(\frac{dr_L}{d\bar{r}_D}\right) =$ sign $\left(\frac{\partial^2 \pi}{\partial r_L \partial r_D}\right)$. We can then conclude

Proposition 3.3 *A ceiling on deposit rates will make lending rates decrease if and only if* $\frac{\partial^2 \pi}{\partial r_L \partial r_D} > 0 \quad \Rightarrow \quad \frac{\partial^2 C}{\partial D \partial L} > 0$. *Therefore, if costs are separable, there will be no effect on* r_L.

3.4 Monopolistic Competition

We now apply the Salop circle model to the banking problem. First we want to find out whether free competition leads to the efficient number of banks.
In the model, there are n banks collecting deposits from the D depositors, each endowed with one unit of cash, and the bank invests yielding a constant return r. There is a transportation cost for the depositor at αx. The total transportation cost will be:

$$2n \int_0^{\frac{1}{2n}} \alpha x D dx = \frac{\alpha D}{4n} \tag{18}$$

The optimal number of banks is fund by minimizing total transportation costs and costs of setting up banks:

$$\frac{\partial \left(nF + \frac{\alpha D}{4n}\right)}{\partial n} = F - \frac{\alpha D}{4n^2} = 0 \quad \Rightarrow \quad n^* = \frac{1}{2}\sqrt{\frac{\alpha D}{F}} \tag{19}$$

If competition is free, the number of banks will be too high. To see this we start by determine the location of the 'marginal depositor', who is indifferent about going to bank i or bank $i+1$. The distance $\hat{x}_i$ between the marginal depositor and the bank is defined by:

$$r_D^i - \alpha \hat{x}_i = r_D^{i+1} - \alpha \left(\frac{1}{n} - \hat{x}_i \right) \quad \Rightarrow \quad \hat{x}_i = \frac{1}{2n} + \frac{r_D^i - r_D^{i+1}}{2\alpha}$$

and the volume of deposits attracted by bank i is

$$D_i = D \left[\frac{1}{n} + \frac{2r_D^i - r_D^{i+1} - r_D^{i-1}}{2\alpha} \right],$$

leading profits to be

$$\pi_i = D(r - r_D^i) \left(\frac{1}{n} + \frac{2r_D^i - r_D^{i+1} - r_D^{i-1}}{2\alpha} \right)$$

Maximizing these expressions for all i gives the symmetric unique solution

$$r_D^i = \ldots = r_D^n = R - \frac{\alpha}{n} \quad \Rightarrow \quad \pi_1 = \ldots = \pi_n = \frac{\alpha D}{n^2}$$

Setting this equal to the setup cost gives

$$n_e = \sqrt{\frac{\alpha D}{F}} \tag{20}$$

This is clearly larger than the efficient number of banks.

3.4.1 Deposit rate regulations revisited

A model analogue to the previously presented is used here with the slight modification that depositors are now also borrowers with an inelastic credit demand L and have utility $U = (1 + r_D) - \alpha x_D - (1 + r_L)L - \beta x_L$. The equilibrium rates become $r_D^e = r - \frac{\alpha}{n}$ and $r_L^e = r + \frac{\beta}{nL}$. This gives profit $\pi^e = \frac{D(\alpha+\beta)}{n^2}$. Again, setting this equal to the entry cost gives

$$n^e = \sqrt{\frac{D(\alpha + \beta)}{F}} \tag{21}$$

There will be independent pricing of deposits and loans, unless the bank is allowed to offer tied-up contracts.

The following proposition, which gives the opposite conclusion than the one obtained from the Monti-Klein model can be proved:

Proposition 3.4 *Under deposit regulation, banks will offer tied-up contracts with lower credit rates than in the unregulated case. So, regulation is effective.*

4 Chapter 4 – The Loan Contract

In a sense, one can say that the overall question in this chapter is whether a standard debt contract is efficient. We specifically dig deeper into three aspects: (1) The symmetric information case, (2) The asymmetric information case (ex post moral hazard – costly state verification), (3) Ex ante moral hazard. The section about renegotiation of debt can be skipped, and the section about the role of collateral is postponed until chapter 5.

4.1 Perfect information

In the case where the cash flow $\tilde{y}$ is actually observable, the contract should reflect the attitudes towards risk for the borrower and the lender, such that

$$R'(y) = \frac{I_B(y - R(y))}{I_B(y - R(y)) + I_L(R(y))}, \tag{22}$$

which is the borrower's fraction of the total absolute risk aversion. The idea when deriving this result is to maximize the utility of the borrower subject to the lender obtaining a certain minimal utility level. The result only holds if limited liability constraints are not binding.

The result is not very satisfactory in the banking context. If the bank is risk neutral, then $R'(y)$ should be close to one. But in a typical debt contract $R(y) = \min[y, R]$. So we need to extend the model with an information asymmetry.

4.2 Ex post moral hazard

This section deals with asymmetric information, particularly the case of ex post moral hazard. Two cases are studied, the case of costly state verification and the case of thereat of termination.

4.2.1 Costly state verification

The previous model is modified such that the cash flow is only observable to the lender if he undertakes an audit at a cost of γ. We shall find a contract that is a direct revelation mechanism specified by

1. A repayment function $R(\hat{y})$.
2. An auditing rule, identified as a set S of reports that will lead to an audit.
3. A penalty function $P(y, \hat{y})$.

The first result is that in order for the contract to be incentive compatible, it must hold that (1) No audit $\Rightarrow$ Constant repayment and (2) Audit $\Rightarrow$ Lower repayment:

$$\forall y \notin S \quad R(y) \equiv R \quad \wedge \forall y \in S \quad R(y) \leq R$$

Contracts fulfilling these expressions will be incentive compatible. If they should also be efficient, it must be that

$$\forall y \in S \quad R(y) = \min[y, R] \quad \wedge \quad S' = \{y | y < R\} \tag{23}$$

The first equation states that in the audit zone, taking into account limited liability and incentive compatibility, the repayment must be maximal. The second equation states that an audit will only take place when reimbursement is less than R – bankruptcy. We can interpret the equations as a standard debt contract. We have therefore the following result:

Proposition 4.1 *If both agents are risk neutral, any efficient incentive compatible debt contract is a standard debt contract.*

Diamond suggests that if auditing costs are infinite (auditing impossible) the outcome of a standard debt contract can still be achieved through the introduction of a nonpecuniary cost $\varphi(y)$, that the lender can inflict on the borrower if the latter does not meet its debt obligations. Now we will still have the standard debt contract, but if the borrower reports $y < R$, then the lender will inflict a nonpecuniary cost on the borrower such that his total cost will be the same as if he had reported $y = R$. See figure 4.2 on page 98.

4.2.2 Threat of termination

Bolton & Scharfstein (1990) study the repeated contract (two periods). If $\tilde{y}$ can only take on two values, a high $\bar{y}$ and a low $\underline{y}$ value, then in the one-shot relationship, the borrower will always report $\underline{y}$. The bank may be willing to renew the loan only if $\bar{y}$ is reported in the first period, since it knows that $\underline{y}$ will evidently be reported in the second. If the gain in the first period is high enough compared to the loss in the second, then this contract will be signed. This is equivalent to

$$\pi \geq 0 \quad \Leftrightarrow \quad R \geq L + \frac{L - \underline{y}}{P[\tilde{y}_1 = \bar{y}]} \tag{24}$$

At the same time it must hold that $R \leq E[\tilde{y}]$. The repayment exists if and only if:

$$L - \underline{y} \leq P[\tilde{y}_1 = \bar{y}](E[\tilde{y}] - L) \tag{25}$$

4.3 Ex ante moral hazard

Here we consider a model in which the return of the investment is continuous, and the distribution is influenced by the borrower's effort e, which at the same time gives disutility to the borrower. We wish to maximize the borrower's utility given an IR constraint for the lender, an IC constraint for the borrower and the limited liability constraint. The result is:

Proposition 4.2 *Given the technical condition monotone likelihood ratio, the optimal repayment is*
$R(y) = 0 \quad for \quad y \geq y^*$
$R(y) = y \quad for \quad y < y^*.$

See figure 4.4, page 110.

5 Chapter 5 – Credit rationing

This chapter focusses on credit rationing, meaning that *some borrower's demand for credit is turned down, even though this borrower is willing to pay all the price and non-price elements of the loan contract.*

5.1 Backward-bending supply

Often in credit rationing theory the property if backward bending supply is taken as given. As soon as this is the case, credit rationing can of course occur – see figure 5.2, p. 140. The crucial question is of course what can cause this property.

This can be a result of **adverse selection**. In the model of **Stiglitz & Weiss (1981)**, borrowers differ only by a risk parameter θ. It is assumed that $E[\pi(y)|\theta]$ is increasing in θ. Therefore, there must exist a value θ^*, for which it holds that $E[\pi(y)|\theta^*] = \bar{\pi}$. The projects with $\theta \in [\theta^*, \bar{\theta}]$ thus apply for credit. Consider what happens if the bank raises the payback demand (interest). There are two effects: (1) For a given θ, the interest payment is higher, and thus profits tend to be higher from this effect. (2) Now θ^* will be higher, so the less risky firms drop out of the market. So all together the effect is ambiguous.

Williamson (1987) set up another model, taking as its starting point the costly state verification model (chapter 4).

5.2 Moral hazard

The model is due to **Bester & Hellwig (1987)**. It considers the usual moral hazard setup with two different technologies B and G: $\pi_G G > \pi_B B \quad \wedge \quad B > G$. The loan contract specifies an amount R to be repaid in case of success. The (IC)-condition for the borrower (in order for him to choose the 'good' technology) is:

$$\pi_G(G - R) \geq \pi_B(B - R) \quad \Leftrightarrow \quad R \leq \hat{R} \equiv \frac{\pi_G G - \pi_B B}{\pi_G - \pi_B} \tag{26}$$

This gives rise to a picture like figure 5.6, p. 149. In order to close the model, it is necessary to introduce the supply of credit. If supply is not infinitely elastic, there will be a

local or global maximum at $R = \hat{R}$. Therefore we will have the case of backward-bending supply just like in the Stiglitz-Weiss model.
(If the lenders are price-takers with infinitely elastic supply there will not be credit rationing and markets will clear.)

5.3 Collateral as sorting device

The model is due to **Bester (1985)**, and suggests using collateral as a discriminating device for the bank to distinguish between borrowers. In the setup, there are only two kinds of borrowers, low-risk θ_a and high-risk θ_b. The idea in the model is much like in **Rothschild & Stiglitz (1976)**, where the equilibrium, if it exists must be a separating equilibrium. However, we may not be sure that it actually exists. Examine figure 5.4, p. 145, which summarizes the whole model. Notice the following:

- Banks obtain zero profits due to competition.
- In figure 5.4, BB' is steeper than AB since an additional unit of collateral costs more to the firms than what the bank obtains from it. Same way with CC' compared to AC.
- Iso-profit curves for the risky firms are above and on the right of those for the type a firm since the former are riskier, and the firms' profits are a convex function of the cash flow they obtain.

6 Chapter 6 – Macro

Not part of the curriculum!

7 Chapter 7 – Bank runs

This chapter studies the phenomenon of bank runs (individual bank) or bank panics (entire sector). The conventional explanation for a bank run is observation of large withdrawals leading to fear of bankruptcy and even more withdrawals.
A central question addressed is whether usual deposit contracts (withdrawal on demand) are efficient, and whether the fractional reserve system is justified.

7.1 Liquidity insurance – revisited

Review the model of chapter 2: Consider a three-period, one good economy with a continuum of ex-ante identical agents, each endowed with one unit of the good. Ex-post, at $t = 1$, the agents learn whether they have the utility function $u(C_1)$ or $\rho u(C_2)$. The ex-ante expected utility is thus $U = \pi_1 u(C_1^1) + \pi_2 \rho u(C_2^2)$, where C_t^i is the consumption of

an agent of type i at time t. It is assumed that $u' > 0$ and $u'' \leq 0$. The good can either be stored (with a perfect storage technology) or invested in a long-run technology which returns $R > 1$ units at $t = 2$, but only $L < 1$ if liquidated at $t = 1$.
The result of the model was, that:

$$\text{Pareto efficient allocation} = \text{Bank} \succsim \text{Market economy} \succsim \text{Autarky}.$$

The fractional reserve banking system. Notice that competition between banks will lead them to offer the optimal deposit contract. The question is whether this system is stable in the sense that the bank will be able to fulfill its obligations.
Notice first that $C_2^* \geq C_1^* \Leftrightarrow \rho R \geq 1$, since the patient consumer can always withdraw deposits at $t = 1$ and store until the next period. Therefore, **if $\rho R < 1$, the optimal allocation can never be implemented through a deposit contract**.
If $\rho R \geq 1$ we have two possibilities:

- If the patient consumers expect the bank to be able to fulfill its obligations, it will be optimal for them to keep their deposits in the bank. The bank will be solvent, and the optimal allocation will be reached.
- If on the other hand the patient consumers expect that the bank will not be solvent. They will then withdraw their deposits, and indeed the bank will fail. The expectations turn out to be self-fulfilling. This is the case of a **bank run**.

Thus, there are two reasons why the efficient allocation may not be reached:

1. If the relative return from period 1 to 2 is too small (worse than the storage technology). This can be extended to the case of financial markets, where the 'storage technology' yields a higher return, and it is therefore 'harder' for a deposit contract to reach the efficient allocation.
2. The other reason is due to the second Nash-equilibrium, where the inefficient case of a bank run is reached because of a lack of confidence in the bank. This can also be seen as the case of lack of coordination between depositors.

The text suggests a row of solution concepts, including **Narrow banking**, **Suspension of convertibility** (a maximum number of withdrawals at $t = 1$), **Deposit insurance system** (e.g. 'Indskydergarantifonden'), and **Equity instead of deposit contracts**.

7.2 Interbank market

This section uses a model of **Bhattacharya & Gale (1987)**, which is a slightly modified version of the previous model of liquidity insurance. Here there are several banks that are confronted with i.i.d. liquidity shocks. The proportion of patient consumers who withdraw early can be π_L or π_H with probabilities p_L and p_H. Therefore the bank can only offer **contingent contracts**, where the depositors bear the liquidity risk. This can be overcome by opening an interbank market, where the solution can be shown to be:

$$C_1^* = \frac{1 - I^*}{\pi_a} \quad C_2^* = \frac{I^* R}{1 - \pi_a} \quad , \pi_a = p_L \pi_L + p_H \pi_H.$$

This is independent of the type of the bank[2], and therefore depositors are completely insured. This can be implemented by an interest rate on the interbank market exactly equalling supply and demand for liquidity.
However, there is a problem with this solution, namely **if the type of the bank is not observable**. In this case the banks will be reporting to be of the same type except from the very special case in which the interest rate on the interbank market exactly equals the interest rate of the deposit contracts $(R - 1)$. **The second-best solution will involve imperfect insurance of depositors**.

Hellwig (1994) extends the model further by introducing a third investment opportunity from periode 1 to 2 with a random return. He shows that both in the efficient allocation and in the second best allocation under asymmetric information, **depositors should bear some interest rate risk**. This is because non-diversifiable risk should be borne by all agents in proportion to their their risk tolerance (c.f. chapter 4)[3].

The chapter ends with a short discussion of the importance of a Lender of Last Resort (LLR) – typically a Central Bank.

8 Chapter 8 – Risk Management

8.1 Credit / Default risk

This subsection holds two different models on credit risk. The first approach assumes that defaults are results of a Poisson proces, while the second uses an option pricing approach.

8.1.1 Defaults as Poisson events

This approach starts out by noting that the expected cost of default risk of a loan consisting of a series of promised payments at future dates can be expressed as

$$P_0 - P = \sum_{k=1}^{n} C_k e^{-rt_k} - \sum_{k=1}^{n} C_k p_k e^{-rt_k},$$

[2]In the case of no interbank market, these expressions will be the same, just with π_a replaced with the bank's type, and it therefore depends on this.

[3]Furthermore, in the 2nd best solution, banks have to provide their depositors at least the rate of return they can obtain on financial markets.

which is the difference between the expected repayment including the risk of default minus the value of the loan if there was no default risk.
The usual way to characterize the credit risk, however, is to calculate the spread, which is $s = R - r$, where R is the yield to maturity of the risky loan:

$$P = \sum_{k=1}^{n} C_k p_k e^{-rt_k} = \sum_{k=1}^{n} C_k e^{-Rt_k} \tag{27}$$

If one assumes that the probabilities in (27) are drawings of a Poisson proces ($p_k = e^{-\lambda t_k}$), one obtains

$$\sum_{k=1}^{n} C_k e^{-Rt_k} = \sum_{k=1}^{n} C_k e^{-\lambda t_k} e^{-rt_k} \quad \Rightarrow \quad R = r + \lambda \quad \Leftrightarrow s = \lambda, \tag{28}$$

such that **the risk spread is independent of the characteristics of the loan**.

8.1.2 Option pricing approach

This section is due to **Merton (1974)**. Here the banks will lend an amount D_0 to a firm promising to repay D at $t = T$. The yield to maturity r_L is defined by

$$D = D_0 e^{r_L T} \Rightarrow \log\left(\frac{D}{D_0}\right) = r_L T \Rightarrow r_L = -\frac{1}{T} \cdot \log\left[\frac{D_0}{D}\right]$$

From this we can calculate an expression for the spread $s = r_L - r$:

$$\begin{aligned} s = -\frac{1}{T} \cdot \log\left[\frac{D_0}{D}\right] - r = -\frac{1}{T} \cdot \log\left[\frac{D_0}{D}\right] - -\frac{1}{T} \cdot \log(e^{rT}) = \\ -\frac{1}{T} \cdot \log\left[\frac{D_0}{D} \cdot e^{rT}\right] = -\frac{1}{T} \cdot \log\left[\frac{D_0}{D \cdot e^{-rT}}\right] \end{aligned} \tag{29}$$

Denote the value of the firms assets by $V(T)$. The terminal payoff to the bank is thus $\min(D, V(T))$. This is similar to buying the firm's assets and selling a call option to the stockholders. In order to proceed with the model, we only need to specify $V(T)$. Following Merton (1974), we assume $V(T)$ to be a geometric Brownian motion. Therefore, we can use the Black-Scholes formula for option pricing, and the value of the loan is:

$$D_0 = VN(h_1) + De^{-rT} N(h_2) \tag{30}$$

Combining (29) with (30) yields

$$s = -\frac{1}{T} \log\left[\frac{N(h_1)}{d} + N(h_2)\right] \quad , d = \frac{De^{-rT}}{V}, \tag{31}$$

where d is interpretable as a 'quasi debt to asset ratio'.
The advantages of the present model compared to the Poisson approach is that (1) Default probability is endogenous, (2) The market pricing of risk is used, and (3) The liquidation value of the firm is not zero.
Interesting comparative statics gives the following

1. s increases with d. (High leverage $\Rightarrow$ high spread)
2. s increases with σ. (High risk $\Rightarrow$ high spread)
3. The global risk premium sT increases with T.

It is fairly simple to extend the model to include collateral.

8.2 Liquidity risk

8.2.1 Reserve Management

This short section uses an 'OR'-alike approach. The question is how large the reserves R should be. Assume that net withdrawals is a random variable $\tilde{x}$. If the realization is greater than R, the bank must pay a penalty interest r_p. The banks profit is thus

$$\Pi(R) = r_L(D - R) + rR - r_p E[\max(0, \tilde{x} - R)]$$

The first order condition to this gives us

$$\begin{aligned} \Pi'(R) = -(r_L - r) - r_p \left(\int_R^\infty (x - R) f(x) dx \right)' = \\ -(r_L - r) + r_p \int_R^\infty f(x) dx = -(r_L - r) + r_p \cdot P[\tilde{x} \geq R] = 0 \quad \Leftrightarrow \\ P[\tilde{x} \geq R] = \frac{r_L - r}{r_p} \end{aligned} \tag{32}$$

8.2.2 Monti-Klein revisited

The setup stays pretty much the same here, and we can write the profit as

$$\Pi(r_L, r_D) = (r_L - r)L(r_L) + (r - r_D)D(r_D) - r_p E[\max(0, \tilde{x} - D(r_D) + L(r_L))]$$

As usual, we characterize the solution by the first order conditions

$$\begin{aligned} \frac{\partial \Pi}{\partial r_L} = (r_L - r)L'(r_L) + L(r_L) - r_p \cdot P[\tilde{x} \geq R]L'(r_L) = 0 \\ \frac{\partial \Pi}{\partial r_D} = (r - r_D)D'(r_D) - D(r_D) + r_p \cdot P[\tilde{x} \geq R]D'(r_D) = 0, \end{aligned}$$

and using elasticities

$$\begin{aligned} r_L^* &= \frac{r + r_p \cdot P[\tilde{x} \geq R]}{1 - \frac{1}{\epsilon_L}} \\ r_D^* &= \frac{r + r_p \cdot P[\tilde{x} \geq R]}{1 + \frac{1}{\epsilon_D}} \end{aligned} \tag{33}$$

One can show that the following holds

1. If r_p increases, then r_L^* and r_D^* also increase, and therefore L decreases while D increases.

2. If the variance of $\tilde{x}$ increases, then if $R > 0$, L decreases.

8.3 Market risk

This section uses the CAPM model. We assume that the investors only care about first and second moments of the distribution of the value of their portfolio. One can show that the first order conditions to the utility maximization problem can be written as

$$-\lambda\rho + Vx = 0 \quad \Leftrightarrow \quad x = \lambda V^{-1}\rho \tag{34}$$

This equation says that all investors should hold collinear portfolios, which are a combination of the riskfree asset and a mutual fund. The individual component is captured by $\lambda = -\frac{\partial U}{\partial \mu}/(2\frac{\partial U}{\partial \sigma^2})$.
The **Pyle, Hart-Jaffee** approach. We view the bank as a portfolio manager with only two risky activities L and D and a risk-free asset. Once again (of course) the banks optimum will be to choose x^* such that $x^* = \lambda V^{-1}\rho$. From this, we obtain the following results:

Proposition 8.1 *If $\bar{r}_D < r < \bar{r}_L$ and $cov(\tilde{r}_L, \tilde{r}_D) > 0$, then $x_L^* > 0$ and $x_D^* < 0$.*
x_L^* is increasing in $(r_L - r)$ and decreasing in $(r_D - r)$ and $var(\tilde{r}_L)$.
$|x_D^*|$ is increasing in $(r_L - r)$ and decreasing in $(r_D - r)$ and $var(\tilde{r}_D)$.

The first part of the proposition can be seen as an explanation for the intermediation activity of banks. The proof of this part of the proposition can pretty simply be made by calculating x^*.
The following critique has been put forward: (1) The fact that all banks should hold collinear portfolios, (2) The existence of a utility function of the bank, and (3) If the possibility of bank failure is taken into account the symmetry between assets and liabilities breaks. The last point is addressed below.

We now apply the model to the issue of capital requirements. We assume that the bank chooses its portfolio of $(n+1)$ securities, of which security 0 is the risk free asset. Liabilities are held fixed, deposits D and equity K. At $t=1$, the bank is liquidated, and the value

$$\tilde{K}_1 = K + \sum_{i=1}^{n} x_i(\tilde{r}_i - r_0)$$

is distributed to the shareholders. The bank behaves as a portfolio manager and seeks to maximize $E[u(\tilde{K}_1)]$. First, we try to characterize the solution in the absence of solvency regulation. Here, the bank will just maximize $E[u(\tilde{K}_1)]$, leading again to $x_1^* = \lambda_1 V^{-1}\rho$. We then examine the probability of default:

$$P[\tilde{K}_1 < 0] = N(-\frac{\mu}{\sigma}) = N\left(-\frac{K + \sum_{i=1}^{n} x_i^* \rho_i}{(x^* \cdot V x^*)^{1/2}}\right).$$

A solvency ratio is usually defined as

$$CR = \frac{K}{\sum_{i=1}^{n} \alpha_i x_i^*}$$

We then conclude the following

Proposition 8.2 *In the absence of solvency regulation, and if banks do not take into account limited liability, the probability of banks' failure is a decreasing function of CR.*

It is a reasonable argument why there should be some minimum restriction on banks CR. If we impose this restriction, the new solution will become

$$x_2^* = V^{-1}[\lambda_2\rho + \nu_2\alpha]$$

So we have the result that

Proposition 8.3 *If α is not collinear to ρ and if the solvency constraint is binding, the bank will choose an inefficient portfolio, where x_2^* is not collinear to $V^{-1}\rho$. However, if the weights α used to compute CR are proportional to the systematic risks β, the solvency regulation becomes efficient, in the sense that all banks choose efficient portfolios and the probability of default decreases.*

9 Chapter 9 – Regulation

The regulatory instruments are normally classified into six types:

1. Deposit interest rate ceilings (chapter 3)
2. Entry, branching, network and merger restrictions (briefly chapter 3)
3. Portfolio restrictions (Points to the discussion of whether universal banking should be allowed. Will not be addressed further).

4. Deposit insurance (Chapter 7, but more to come...).
5. Capital requirements (Chapter 8, but more to come...).
6. Regulatory monitoring – mainly a question of closure policy (To come...).

9.1 Deposit insurance

I chapter 7 it was shown how deposit insurance could be the solution to the problem of bank runs. Here, we study the issue of deposit insurance further.
First, we establish that the existence of a deposit insurance system creates a moral hazard problem for the bank. Note that the liquidation value of the bank in a two-period model can be written as:

$$\tilde{V} = \tilde{L} - D + \max(0, D - \tilde{L}) = F + (\tilde{L} - L) + [\max(0, D - \tilde{L}) - P].$$

Assume furthermore that $\tilde{L}$ can take only two values: X with probability θ and 0 with probability $1 - \theta$. The expected profit for the stockholders is then

$$\pi \equiv E(\tilde{V}) - F = (\theta X - L) + ((1 - \theta)D - P). \tag{35}$$

One can see the moral hazard problem arise from equation (35). For a fixed P, the bank will, within a class of projects with the same NPV, choose those with the lowest θ (highest risk). **So, a flat deposit insurance pricing is problematic since it creates moral hazard.**

The question that must follow is how insurance premiums then should be priced. If we assume that the value of the bank's assets follow a geometric random walk, we can use option pricing to answer this question, since we can invoke the Black-Scholes formula. The idea is that **the deposit insurance payment is exactly identical to a put option on a bank's assets a strike price** D, and was put forward by **Merton (1977)**. The no arbitrage price of deposit insurance is $P^* = De^{(r_D - r)T} N(h_2) - LN(h_1)$. One can show from the expression and the expressions for h_1 and h_2 that $\frac{P^*}{D}$ is not constant, namely it should hold that

Proposition 9.1 *The actuarial rate $\frac{P^*}{D}$ of deposit insurance is an increasing function of the deposit to asset ratio $\frac{D}{L}$ and of the volatility σ of the bank's assets.*

Chan, Greenbaum & Thakor (1992) have stated that fairly priced deposit insurance may not be possible due to the existence of asymmetric information. Suppose therefore that θ is private information for the bank. Fairly priced deposit insurance is possible if there exists a $P(D)$ such that

$$P(D(\theta)) = (1 - \theta)D(\theta), \tag{36}$$

where $D(\theta) = \arg\max_D \Pi(D,\theta) = \arg\max_D(\theta X - L) + (1-\theta)D - P(D(\theta))$. This will require that

$$\frac{\partial \Pi}{\partial D}(D(\theta),\theta) = (1-\theta) - P'(D(\theta)) = 0 \tag{37}$$

If we differentiate (36) with respect to θ we get

$$P'(D(\theta))D'(\theta) = (1-\theta)D'(\theta) - D(\theta). \tag{38}$$

If we compare (37) and (38) we see that it must be that $D(\theta) = 0$, which is absurd. Therefore, **fairly priced deposit insurance is not possible because of asymmetric information**.

9.2 Solvency regulation

We only study one approach here – namely the **Dewatripont-Tirole (1994)** model. It is a principal-agent model with an incomplete contract element. The model consists of 3 periods. In period 0 the balance sheet of the bank is given. The manager of the bank can improve the quality of the loans by exerting effort, but at a personal cost. In period 1, a first repayment v is obtained and at the same time a signal u is observed about the future liquidation value η at date 2. u and v are independent, but are both related to the level of effort. The idea is for the depositors (the regulator) to choose a rule for reorganizing the bank such that the manager has incentives to choose high effort level.
Under **complete information** the matters are very simple; the bank (manager) should be allowed to continue if u is higher than some threshold value $\hat{u}$. So, here the optimal decision rule only depends on u.
Under **incomplete information** matters are more complex. Here the decision rule depends on both the signals u and v. One can show that the rule will give rise to a decreasing function as shown in figure 9.1, p. 279, where the shaded areas correspond to ex-post inefficiency.

9.3 Closure of a bank

9.3.1 Who should decide?

Repullo (1993) examines who should decide whether the bank should be liquidated or not. In this tree period model, the bank's investment returns $\tilde{R}$ in period 2, but can be liquidated in period 1, returning $\frac{1}{2} < L < 1$. Assume that $E[\tilde{R}|u] = u$. Thus the first-best decision rule is to close the bank if $u < L$. However since u is not verifiable, we have to delegate the decision. Repullo examines the two possibilities of a central bank or a deposit insurance fund. He finds the following:

Proposition 9.2 *The optimal allocation of the decision to close banks is to grant this power to the Central Bank when withdrawals are small ($v < \hat{v}$) and to the deposit insurance* *fund when they are large ($v > \hat{n}$).*

9.3.2 Closure – when?

This model is due to **Mailath & Mester (1994)**. The idea is that we have a three-period game, where the bank, having obtained one unit of deposits, chooses twice between a safe investment (net return r) and a risky investment that yields at NPV at $p(1+\rho) < 1+r$, but $\rho > r$. In between the two investments, the regulator can decide to close the bank (at cost C) or let it continue. Two cases arise:

1. The bank makes the valuation that $(R, S) \sim (S, R) \succ (R, R)$. Here the equilibrium is always (R, S) for the bank and *Continue* for the regulator.

2. The bank makes the valuation that $(R, R) \succeq (R, S)$. Here the equilibrium is more complicated. Notice that if the bank is left to continue, it will always choose R in the second period. If the bank plays R in the first period, the bank should be closed, but if the closure cost C or the probability of success is sufficiently high, than closing is not credible. For some values, the threat is credible, and this leads the bank to choose (S, R).

Have a look at figure 9.3, p. 284 and figure 9.4, p. 286 which explain the outcome of the model.